Entanglement swapping and the foundations of physics

Benedict Wren

Abstract

The interpretation of quantum mechanics still causes debate despite almost all empirical evidence ruling against a local realist viewpoint. There remain loopholes in all experimental tests to date which might still explain the seemingly nonlocal correlations between distant particles. Entanglement swapping, the teleportation of an entangled state, can be used to entangle particles separated in both space and time. This opens up new ways to test Bell's theorem. I present the physics of entanglement swapping and its key process, the Bell state measurement - the entangling of independently created photons depends on their being indistinguishable from each other. I assess the role of entanglement swapping in testing Bell's theorem and discuss ways of closing the detector and locality loopholes. The timing of the Bell state measurement can be delayed to allow a delayed-choice version of entanglement swapping. This leads to the seemingly paradoxical situation that entanglement can be projected onto photons that have already been detected. I discuss a possible explanation for this paradox. I propose a test of Bell's inequality using delayed-choice and discuss a recent related experiment where Bell's inequality was violated by photons entangled in time.

Contents:

List of figures

List of tables

Introduction

Entanglement, a phenomenon at the heart of quantum mechanics, presents conceptual problems for everyday notions such as space and time. It occurs in certain atomic transitions and may also be 'produced' by suitable interactions between particles. In both cases, the entangled particles share a common history. Yet another surprising feature of quantum mechanics enables independent particles to be entangled [1]. This is the basis of the Bell state measurement [2] that projects two indistinguishable particles onto an entangled state and is the key principle behind quantum teleportation [3]. Entanglement swapping [3,4] is a generalisation of this protocol. Rather than the state of a single particle being transferred to another particle, the entangled state itself can be transferred to two or more particles, however remote and seemingly instantaneously.

As well as applications in the field of quantum information [5], it will play an important role in exploring the foundations of quantum mechanics, particularly in testing the assumptions of locality and realism underpinning local hidden-variables (LHV) theories [6]. For the first time it allows us to test Bell's theorem using particles that share no common history and which have different sets of local hidden variables. This may lead to greater conflicts between local realism and quantum mechanics [5]. Greenberger et al. [7] have derived a theorem, similar to their GHZ argument [8] that shows that entanglement swapping can be used to demonstrate inconsistencies between LHV theories and quantum mechanics without the need for inequalities. Furthermore this theorem is not constrained by the detector loophole [9]. This opens up the exciting possibility of an 'ultimate Bell type test' which could close all the possible loopholes exploitable by a LHV theory in a single experiment. Entanglement swapping can also be used to create multiple-particle entanglement, such as the three-particle GHZ states. The correlations between which can have demonstrated violations of local realism in a definitive way without resorting to the statistical argument of Bell's theorem [10].

Remarkably, entanglement can be projected onto particles that have already been measured and have ceased to exist. The temporal order of measurements performed in an experiment is irrelevant. This allows delayed-choice versions of ES, first proposed by [11] and demonstrated by [12]. This opens up a whole new empirical framework for exploring nonlocal interpretations where separation in time is also a factor. It provides an intriguing possibility for testing Bell's theory using pairs of particles where both particles have already been observed but the choice of whether to entangle them has yet to be made. An interesting variation on this theme is to project entanglement onto two particles, one of which has already been observed so that the two particles do not coexist [13].

This work focuses on polarisation-entangled two-photon states where there has been considerable research over the last two decades.

Part 1 – Basic concepts

1.1 Quantum states and entanglement

When we measure a photon's polarisation in a given direction, we obtain one of only two possible outcomes: horizontally polarised (H) or vertically polarised (V). After measurement, the photon is correspondingly in one of two well-defined states $|\mathrm{H}\rangle$ or $|\mathrm{V}\rangle$. Before a measurement, however, a photon's polarisation is undefined and may be represented by a linear superposition of the states $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$, weighted by the complex coefficients α and β as follows.

$$|\Psi\rangle = \alpha|\mathrm{H}\rangle + \beta|\mathrm{V}\rangle$$

This can be represented by a vector in a vector space spanned by the basis vectors $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ which are orthogonal to each other (see Figure 1). Provided that the normalisation condition $|\alpha|^2 + |\beta|^2 = 1$ holds, $|\alpha|^2$ and $|\beta|^2$ represent the probabilities of observing the photon to have H or V polarised light respectively [14].

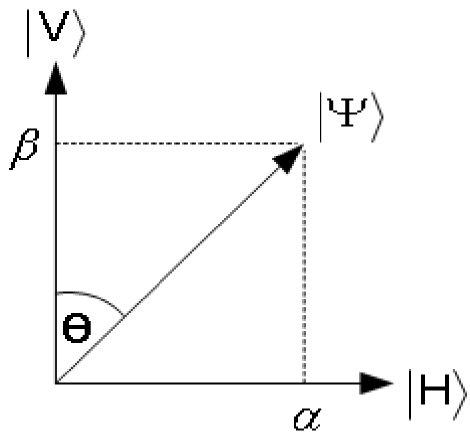

Figure 1 – The quantum state of a single photon is represented by a vector in a space spanned by the orthogonal basis vectors $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$.

A multiple-particle system that is not entangled is represented by a product [15], e.g. for a system comprising of photons 1 and 2 in the states $|\Psi\rangle_1$ and $|\Psi\rangle_2$ respectively, the overall state is

$$|\Psi\rangle_1|\Psi\rangle_2 = \left(\alpha_1|\mathrm{H}\rangle_1 + \beta_1|\mathrm{V}\rangle_1\right)\left(\alpha_2|\mathrm{H}\rangle_2 + \beta_2|\mathrm{V}\rangle_2\right) \quad (1.1)$$

An entangled state *cannot* be expressed as a product of states each representing an individual particle [16] but remains a linear superposition of such product states (i.e. it is not factorisable). Two photons may have their polarisations entangled in one of the four following states, known as the Bell states [2]:

$$\left|\Psi^{+}\right\rangle = \frac{1}{\sqrt{2}}(\left|H\right\rangle_1 \left|V\right\rangle_2 + \left|V\right\rangle_1 \left|H\right\rangle_2) \quad (1.2)$$

$$\left|\Psi^{-}\right\rangle = \frac{1}{\sqrt{2}}(\left|H\right\rangle_1 \left|V\right\rangle_2 - \left|V\right\rangle_1 \left|H\right\rangle_2) \quad (1.3)$$

$$\left|\Phi^{+}\right\rangle = \frac{1}{\sqrt{2}}(\left|H\right\rangle_1 \left|H\right\rangle_2 + \left|V\right\rangle_1 \left|V\right\rangle_2) \quad (1.4)$$

$$\left|\Phi^{-}\right\rangle = \frac{1}{\sqrt{2}}(\left|H\right\rangle_1 \left|H\right\rangle_2 - \left|V\right\rangle_1 \left|V\right\rangle_2) \quad (1.5)$$

The polarisation of a given photon is undefined; however the correlation between them is fully determined. Consider $\left|\Psi^{-}\right\rangle$ which is known as the singlet state. Measurement of one of the photons causes the collapse of the wave function into either $\left|H\right\rangle_1 \left|V\right\rangle_2$ or $\left|H\right\rangle_1 \left|V\right\rangle_2$ whereupon the polarisation of the other is immediately known. If photon 1 is observed to have horizontal polarisation then photon 2 must have vertical polarisation and vice versa.

Certain types of atomic transition emit pairs of entangled photons. An example is in spontaneous parametric down conversion (SPDC) that occurs inside a suitable nonlinear crystal such as BBO or LBO (beta-barium borate or lithium triborate) when pumped by a laser of suitable frequency. There are two types of SPDC: Type I emits photons with parallel polarisations (i.e. the $\left|\Phi^{\pm}\right\rangle$ states) whilst Type II emits photons with antiparallel polarisations (the $\left|\Psi^{\pm}\right\rangle$ states) [2].

1.2 The EPR experiment and Bell's theorem

The famous EPR thought-experiment derives its name from the authors (Einstein Podolsky and Rosen) of a 1935 paper in which they contended that the wave function description of a system in quantum mechanics must be incomplete [17]. They argued that if one could make a measurement on a system without disturbing it then the result of that observation must correspond to an element of physical reality (a real property of the system) that a complete theory should explain. EPR considered entangled systems of two particles and correlations of position and momentum between them. However, their argument also applies to polarisation entanglement.

Consider a pair of photons entangled in the singlet state which we know means that, in a given direction, their polarisations are anticorrelated (opposite). If we perform a measurement and deduce that photon 1is in the $\left|V\right\rangle_1$ state then surely we would know, without needing to measure photon 2, that it is in the $\left|H\right\rangle_2$ state. Therefore photon 2 must have been horizontally polarised *before* it was measured and this must correspond to a real physical property of the photon. This type of logical argument leads to contradictions with quantum mechanics which posits that before measurement, the polarisation of each photon is undefined. Central to the EPR argument are two assumptions:

(1) Realism – Observables of a quantum system such as H and V polarisation correspond to real, physical properties of that system.

(2) Locality – The outcome of a measurement of one particle cannot influence the outcome of a measurement of its entangled partner by means of a signal travelling at or less than the speed of light.

Theories based on these assumptions are known as local hidden-variables (LHV) theories. Using an argument first given by John Bell [6] but here in the context of polarisation rather than electron spin, I will demonstrate that any LHV theory is in conflict with quantum mechanics and that this conflict leads to an experimentally testable theorem with far-reaching implications.

A large number of pairs of photons with their polarisations entangled in the $|\Psi^-\rangle$ state are sent consecutively in opposite directions towards the two scientists Alice and Bob (see Figure 1). They each measure the polarisation in one of two directions (α and α' for Alice, and β and β' for Bob) by rotating the axis of their respective polarising beam splitters (PBS). Each scientist chooses one of these two bases (H_α/V_α or the $H_{\alpha'}/V_{\alpha'}$ for Alice, and H_β/V_β or $H_{\beta'}/V_{\beta'}$ for Bob) at random and the outcomes at each detector are recorded.

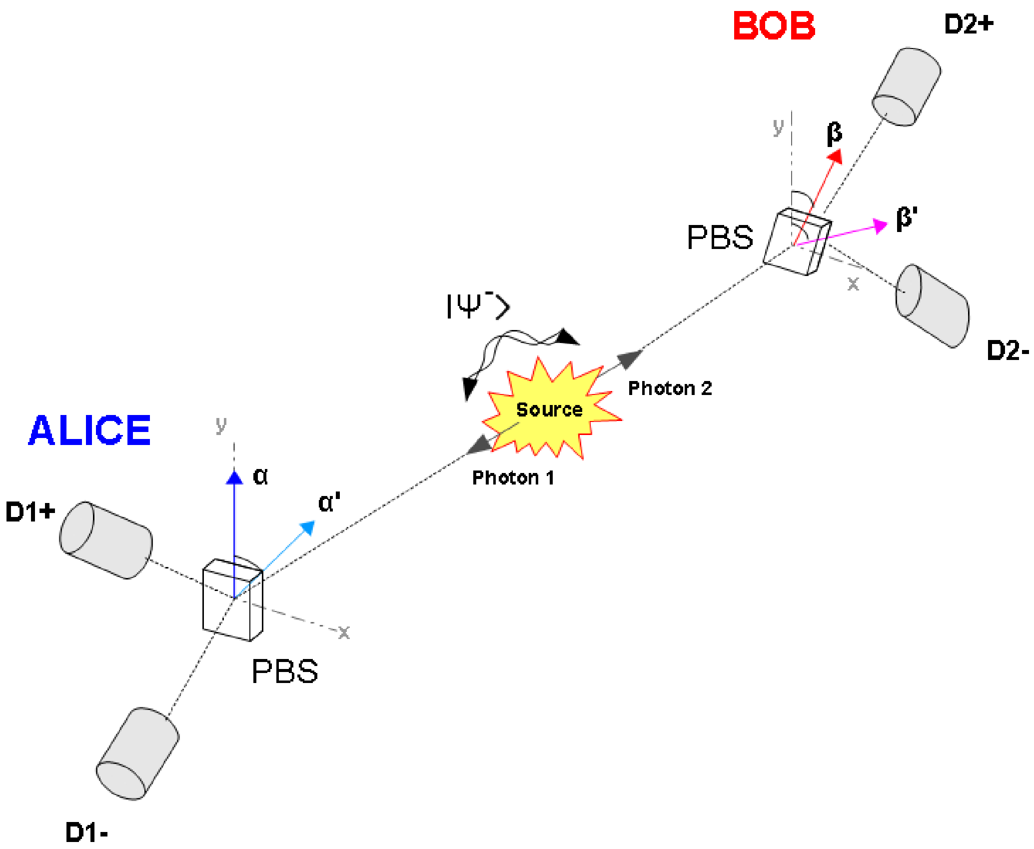

Figure 2 – The EPR experiment. Alice receives a photon entangled with Bob's photon. When they each measure polarisations in bases rotated with respect to each other, the predictions of quantum mechanics and local realism diverge and can be used to test Bell's theorem.

Alice and Bob then communicate their results to each other. For each of the four combinations of bases they calculate the correlation coefficient $C(\alpha,\beta)$, which is the number of measurements where they both observe vertically (labelled by +) or horizontally (labelled by -) polarisation in their respective bases minus the number where they disagree on this polarisation, all divided by the total number of observations in the subset defined by their chosen bases [14], i.e.

$$C(\alpha,\beta) = (N_{++} + N_{--} - N_{+-} - N_{-+}) / N_{\alpha\beta}$$

Following the observation of many pairs, this should equal the probability of agreement minus the probability of disagreement, i.e.

$$C(\alpha,\beta) = P_{++} + P_{--} - P_{+-} - P_{-+} \tag{1.6}$$

For each of the four basis combinations, they calculate the Bell parameter [18]

$$S = C(\alpha,\beta) - C(\alpha,\beta') + C(\alpha',\beta) + C(\alpha',\beta')$$

It can be shown that based on assumptions (1) and (2), this value should satisfy the CHSH inequality[1] [19]

$$-2 \le S_{LHV} \le 2 \tag{1.7}$$

The subscript LHV indicates the Bell parameter according to any LHV theory, to differentiate it from the Bell parameter from the experimental result which we can label $S_{\exp}$.

However, if one uses the quantum mechanical expressions for the probabilities in equation (1.6), i.e. $P_{++} = P_{--} = \sin^2(\hat{a} - \hat{b})$ and $P_{+-} = P_{-+} = \cos^2(\hat{a} - \hat{b})$ [20] then, after using a suitable trigonometric identity, quantum mechanics predicts

$$S_{QM} = -\cos 2(\alpha - \beta) + \cos 2(\alpha - \beta') - \cos 2(\alpha' - \beta) - \cos 2(\alpha' - \beta')$$

For certain pairs of angles, this expression violates(1.7). As an example, consider the case where Alice and Bob's polariser axes are oriented as shown in Figure 3, with each axis being separated by an angle ϕ from the next. This expression then reduces to

$$S_{QM}(\phi) = -3\cos 2\phi + \cos 6\phi \tag{1.8}$$

[1] This inequality belongs to a set of inequalities sometimes referred to as Bell's inequalities that are each derived to test local realism but use slightly different auxiliary assumptions that take Bell's original inequality [6] and make it testable by experiment. Such assumptions relate to the efficiencies of photon detectors and the influence on the probability of detection of different polariser settings (see discussion of the Detector Loophole below) [9].

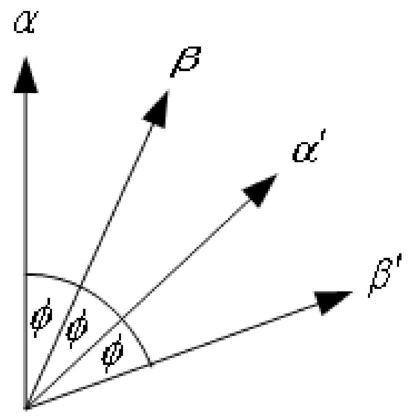

Figure 3 – A possible choice for Alice and Bob's axes that leads to equation (1.8) and the function in Figure 4

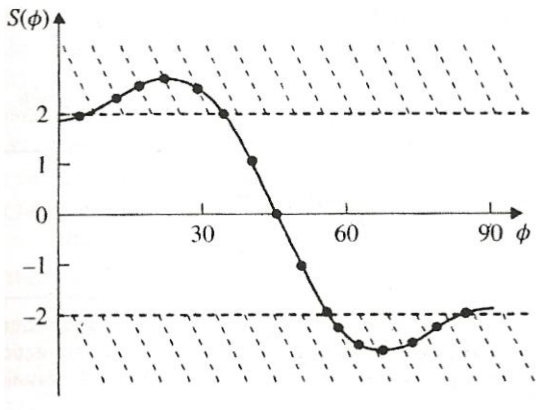

Figure 4 - The behaviour of S_{QM} as a function of the angle between Alice and Bob's pair of axes as shown in Figure 2. The dashed region is where the CHSH inequality is violated.

S_{QM} is plotted in Figure 4 where we see that for certain ranges of ϕ, the CHSH inequality is violated by quantum mechanics. A common choice of axes that maximises the predicted violation is $\alpha = 0°, \alpha' = 45°, \beta = 22.5°$ and $\beta' = 67.5°$ which results in $|S_{QM}| = 2\sqrt{2}$ [22] . By choosing their axes directions appropriately, and obtaining an experimental value $S_{\exp}$ for the Bell parameter, Alice and Bob may test *Bell's theorem* which can be stated as [23]:

No local hidden variables theory can reproduce the predictions of quantum mechanics

Bell's theorem has been tested numerous times [22, 24, 25, 26, 27], with the results strongly in favour of quantum mechanics. This implies that we must give up one or both of the assumptions of realism or locality [14]. However, each of these experiments was open to one or more of the following objections - loopholes in the experimental design that might account for the violation of CHSH yet still allow for a LHV interpretation.

1.2.1 Loopholes in the EPR experiment

1. Locality loophole (*Einstein separability* [28]) – Unless Alice and Bob make their joint measurements within a sufficiently small time interval, depending on their spatial separation, a signal travelling at or below the speed of light could potentially travel from one detector to the other and influence the outcome of the measurement. In other words, to rule out this possibility, Alice and Bob's *measurement events* must be *spacelike separated* [18].

2. Locality loophole (*Strict Einstein separability*) – If either of the polariser settings had been set in advance of the emission of the photons then this would allow extra time for a signal to travel between Alice and Bob's sides of the experiment. To avoid this loophole, the settings of the polarisers need to be changed during the flight of the photons [28]. In other words the *choice events* of the polariser settings must be *spacelike separated* [6].

3. Detector loophole – In order for $S_{\exp}$ to violate the CHSH inequality the detector efficiency must exceed 84% [14]. To circumvent this problem, CHSH employed a 'fair-sampling' assumption which specifies that the probability of detection of each photon is independent of the individual polariser settings [9] even when the polariser is removed. In other words the detected pairs should represent a fair sample of all pairs.

1.3 Measures used in the study of entanglement

I now present some definitions and concepts that will be used throughout this dissertation.

1.3.1 Coherence time

The coherence time τ_c is a measure of the time interval over which entanglement correlations persist]. It is approximately equal to the reciprocal of the bandwidth of the down-converted photons [30] which can cover a wide range of frequencies [31].

$$\tau_c \approx \frac{1}{\Delta\omega} \tag{1.9}$$

1.3.2 Visibility

The visibility V quantifies the degree of interference between two photons. It is given by the difference between (the average values of) the maximum and minimum intensity of the interference fringes divided by their sum [32,33].

$$V = \frac{\langle I \rangle_{\max} - \langle I \rangle_{\min}}{\langle I \rangle_{\max} + \langle I \rangle_{\min}} \qquad 0 \le V \le 1$$

V is used to quantify entanglement and can be calculated from the variation of coincidence rates of two photon detectors firing as a parameter such as the polarisation basis is varied by rotation [4] (see Figure 5).

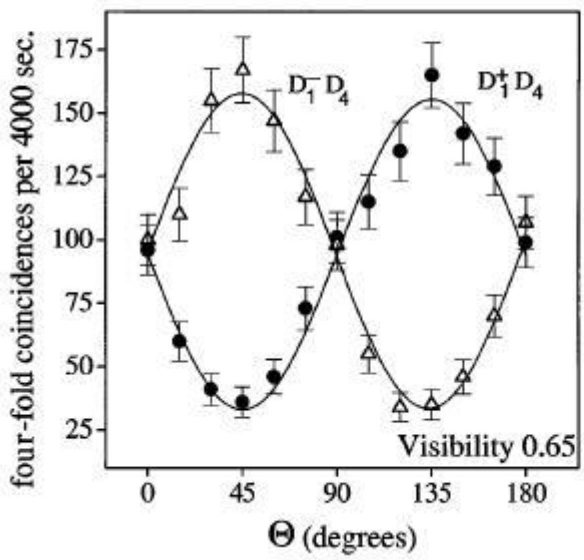

Figure 5 - Visibility can be calculated from the coincidence counts of the detectors in the entanglement swapping experiment.

1.3.3 Fidelity

The fidelity F (where $0 \le F \le 1$) measures how 'close' one quantum state is to another. It can also be interpreted in terms of probabilities [34]. For example, in the entanglement swapping experiment the fidelity is the probability that the measured state is one of the Bell states. The fidelity of an entangled pair of photons is calculated by comparing the observed (e.g.) polarisations of the two photons (in the form of a density matrix) with the theoretically expected polarisations [13, 35].

Fidelity is related to the visibility by the expression [35], [36]

$$F = \frac{1}{4}(3V + 1) \qquad 0 \le F \le 1$$

* * *

It can be shown that two photons are entangled when $V > 0.33$, or equivalently $F > 0.5$ [36, 37] and in order to observe a violation of the CHSH inequality we must have $V > 0.71$, or equivalently $F > 0.78$ [31] .

1.4 Summary of test criteria

It will be useful in the discussion to follow to summarise some key test criteria in Table 1.

Criterion	Criterion	Description
$\mathbb{A}$	$S_{\exp} > 2$	LHV theories must be rejected.
$\mathbb{B}1$	$F > 0.78\ (V > 0.71)$	A violation of CHSH is possible.
$\mathbb{B}2$	$F > 0.5\ \ (V > 0.33)$	Entanglement demonstrated.

Table 1 – Test criteria for entanglement

Part 2 – Entanglement swapping

2. 1 Theory

The basic principle of entanglement swapping is shown in Figure 6A. Two sources, S1 and S2, each emit a pair of entangled photons. Although we could use any entangled state, let's assume that they are both in the $\left|\Psi^-\right\rangle$ state of equation (1.3). Photon 1 is sent to Alice and photon 4 is sent to Bob. The combined state of the system is therefore given by their product

$$\left|\Psi\right\rangle_{1234} = \left|\Psi^-\right\rangle_{12}\left|\Psi^-\right\rangle_{34}$$

This can be re-expressed as a linear superposition of the Bell states (1.2)-(1.5) of photons 2 and 3 [18]

$$\left|\Psi\right\rangle_{1234} = \frac{1}{2}\left(\left|\Psi^+\right\rangle_{14}\left|\Psi^+\right\rangle_{23} - \left|\Psi^-\right\rangle_{14}\left|\Psi^-\right\rangle_{23} - \left|\Phi^+\right\rangle_{14}\left|\Phi^+\right\rangle_{23} + \left|\Phi^-\right\rangle_{14}\left|\Phi^-\right\rangle_{23}\right) \qquad (2.1)$$

If Victor, say, performs a certain type of procedure known as a Bell state measurement (BSM) on photons 2 and 3, he can project them onto one of the four Bell states with an equal probability of 1/4. The combined state of (2.1) collapses onto one of the four product states contained within the brackets. Victor's photons are destroyed in the BSM but Alice and Bob's photons have become entangled in a way that is apparently instantaneous and nonlocal. This is *entanglement swapping* or the *teleportation of entanglement* [31].

2.2 Delayed-choice

Curiously, the timing of the BSM is irrelevant to the outcome. Alice and Bob's photons may already have been measured and thus destroyed before Victor's BSM projects them onto an entangled state (see Figure 6B). This allows a delayed-choice[2] version of entanglement swapping which is still described by equation (2.1). Following measurements on a series of double pairs, Victor communicates his choice of measurement for each to Alice and Bob who organise their data into subsets dependent on his choice and the outcome of his BSM. The subsets defined by projection of an entangled state will demonstrate correlations consistent with the relevant Bell state [11].

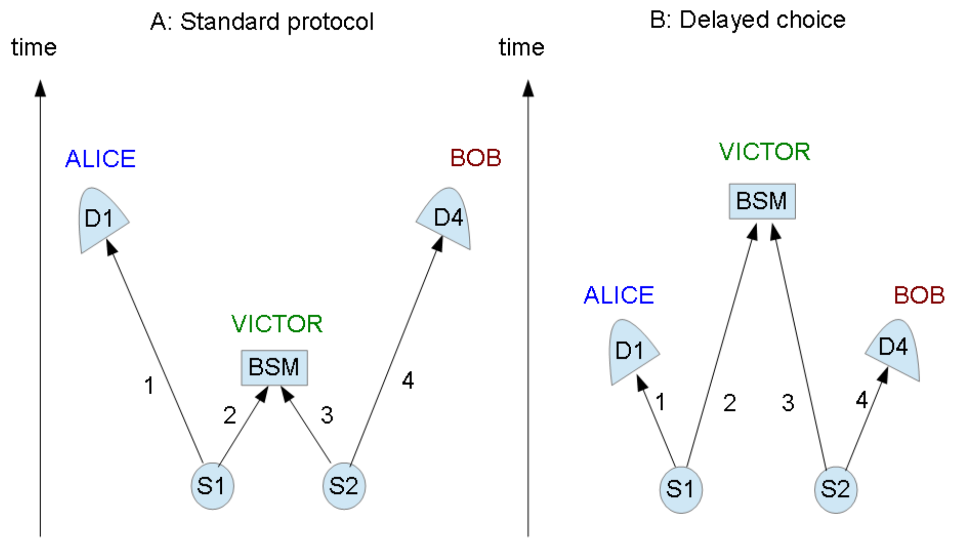

Figure 6 – A: The standard entanglement swapping protocol. Two sources S1 and S2 emit entangled pairs. Victor receives one photon from each pair and projects them onto an entangled state by virtue of a Bell state measurement. This is the key mechanism that projects entanglement onto Alice and Bob's photons. B: In the delayed-choice protocol, Victor may choose to perform the BSM after Alice and Bob have already measured their photons. Curiously, entanglement is still projected onto them and the correlations can be demonstrated.

2.3 Experimental design

A common method of performing entanglement swapping is shown in Figure 7 [4, 18]. A short duration UV pulse is sent through a nonlinear crystal (such as BBO) and an entangled pair of photons (1 and 2) is created by SPDC in the $|\Psi^-\rangle$ state. The pulse travels a short distance and is reflected in the opposite direction through the crystal to create a second entangled pair (photons 3 and 4) also in the singlet state. Photons 2 and 3 are sent to Victor to perform a BSM and project them onto one of the four Bell states(1.2)-(1.5).

[2] Delayed-choice can also be applied to the double-slit experiment where the choice to observe a particle or a wave is not made until the photon has moved beyond the double-slit [57].

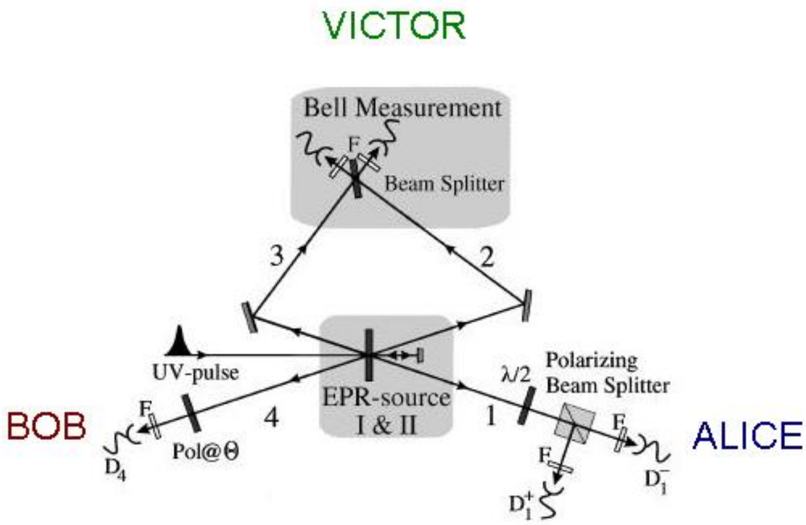

Figure 7 - The experimental set-up: A single UV pulse travels through the same crystal twice which emits two pairs of entangled photons in a very short time frame.

There are various ways of performing the BSM. A *complete BSM* which identifies all four of the Bell states using linear optics is quite challenging. It can be done, but only at the expense of a low efficiency [38]. The method of BSM represented in Figures 7 and 8 is arguably the simplest and was used by [4]. This is a *partial BSM* that identifies only the singlet state. By virtue of this state's antisymmetric nature, photons must emerge from both sides of the beam splitter (BS) [2]. This occurs with a probability of 1/4. The other 3/4 of cases represent the other three Bell states and where the two photons must emerge on the same side of the beam splitter. This means that the other Bell states cannot be distinguished. The efficiency is therefore only 1/4. This low efficiency does not reduce the fidelity of the teleported state [18], but it does increase the duration of the experiment as 3/4 of pairs will need to be discarded.

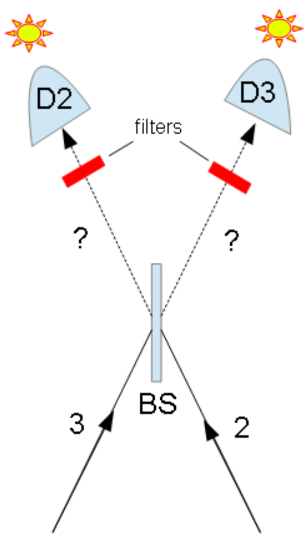

Figure 8 - The Bell state measurement requires both photons to be made indistinguishable. This is done by erasing which-way information such as energy and timing differences.

The BSM involves overlapping the paths of each photon spatially and temporally at a beam splitter (BS) and causing them to interfere. This requires that the two photons are made indistinguishable. Great care must be taken to ensure that any information that might differentiate them (their which-way information) is erased [2, 4],(see Figure 8). The greater the degree to which the photons are made indistinguishable, the higher the visibility of the interference in the BSM, and correspondingly the higher the fidelity of the teleported state [31].

The main contributions to which-way information come from:

(i) The time difference in which photons 2 and 3 reach the detectors can, in principle, reveal the identity of either photon. To erase timing distinguishability the pairs must either be *created* or *detected* in a time interval much shorter than the coherence time $(\Delta t \approx 0.1\tau_c)$ [39]. The latter method is the most technically challenging due to limitations on the time resolution of photon detectors [39]. Most experimental setups therefore aim to create the pairs within the subcoherence time window, for example by using the reflection method of Figure 7.

(ii) The energy of each photon. Radiation from SPDC covers a wide bandwidth of energies [4]. The energies of the other two photons can therefore reveal the identities of photons 2 and 3.

Narrow bandwidth filters placed in front of each photon detector both increase the coherence time (see equation (1.9)) and preselect photons closely matched in energy. The drawback to using very narrowband filters is a decrease in efficiency and an increase in the duration of the experiment [40].

IF this is successful, the BSM projects Victor's photons onto the $\left|\Psi^-\right\rangle$ state and, as described by equation (2.1), also projects Alice and Bob's photons onto the singlet state. Our two scientists can use an ensemble of such pairs to perform the EPR experiment as described in Section 1.2 (see Figure 2) obtaining a value for the Bell parameter in order to test Bell's theorem.

Part 3 – Entanglement swapping and the foundations of physics

3.1 Testing Bell's theorem

One of entanglement swapping's first proposed uses was in the role of 'event-ready detectors' which would allow the EPR experiment to be performed with photons whose entangled state had been preselected (i.e. before detection) as opposed to postselected (after detection) [9, 31, 41]. This would enable the true number of pairs to be counted regardless of detector efficiency and provide a way around the detector loophole without the need for auxiliary assumptions such as fair sampling [42]. However [43, 44] identified an issue related to the suitability of SPDC sources in relation to the BSM process – the probability of both sources emitting one entangled pair is the same as one source emitting two entangled pairs. A coincidence between D2 and D3 does not then uniquely identify the singlet state. Coincident detections at D1 and D4 (see Figure 7) may still be used to postselect the teleported pairs and thus the loss of fidelity instead becomes a loss of efficiency [43, 45], however this reintroduces another fair sampling assumption of the kind we are trying to eliminate.

Despite this setback, entanglement swapping can be used to falsify the CHSH inequality (criterion $\mathbb{A}$) and test Bell's theorem albeit with this auxiliary assumption still needed. The early entanglement swapping experiments lead by Pan [4, 46], experienced problems with visibility and stability of the laser pump that prevented a test of local realism. However in 2002 Jennewein et al. using filters of a narrower bandwidth to increase the coherence time, reported a high fidelity of 0.92 [18]. This allowed a test of CHSH to be performed. Their Bell parameter was $S_{\text{exp}} = 2.421 \pm 0.091$ which violates the CHSH inequality by 4.6 standard deviations. This strongly refutes local realism with a similar significance level as in early falsifications of Bell's inequality such as [24] and [22], both of which were comparable in terms of needing to make the fair sampling assumption and leaving open the stronger of the locality loopholes by not ensuring *Strict Einstein Separability*.

Another possible criticism of Jennewein's experiment was that their scheme (essentially the same as in Figure 7) did not use completely independent sources because the same pulse is used to twice excite a single crystal. Independent sources should constrain the photons to have different values of any hypothesised hidden variables [5]. This is predicted to place tighter restrictions on a LHV theory and allows for new ways to test Bell's theorem which do not rely on inequalities and which are not limited by detector efficiency [7, 47].

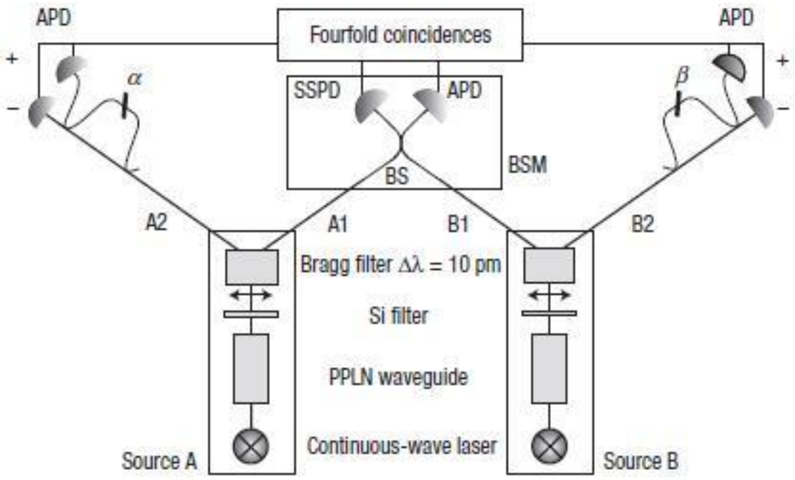

Figure 9 - Fully independent sources lead to tighter restrictions on local hidden variables.

Full independence between sources was achieved by Halder et al. [48] using the creation times of the photons rather than their detection times to select pairs (see Section 2.3 and Figure 9). This required extremely narrowband filters and ultrafast photon detectors which resulted in very low count rates making it impractical to take the extra measurements needed to test CHSH. However, a visibility of 0.77 was achieved which met criterion $\mathbb{B}_1$ and would have allowed a violation of CHSH. Improvements in detector efficiency would increase the fourfold coincidence rate to an acceptable level and allow a test of CHSH [40].

Quasi-independent sources were implemented by Yang et al. [48] and Kaltenbaek et al. [33]. Both methods needed some form of time synchronisation of the laser pulses to create the photon pairs. Yang's team used an optical method that involved coupling two pulses in a Ti:sapphire crystal. Kaltenbaek's team used a method based on electronic feedback. Their Bell parameters were 2.308 ± 0.095 and 2.40 ± 0.09 respectively. Both were large violations of the CHSH inequality of between three and four standard deviations. However, since both methods rely on some kind of interaction between the two laser pulses, then both could in principle be described by a single hidden variable.

All of these experiments left open the locality loophole by not ensuring *Strict Einstein separability* between the 'choice' events of the polariser settings. At least in theory, this would allow a signal travelling at the speed of light or less to travel between the polarisation analysers (PBS) in the time interval in which they had been set, and somehow influencing the outcome in the other wing of the experiment. Jennewein's paper was the only one discussed so far to specifically mention that the detection events were spacelike separated (*Einstein separability*) ruling out the less strict form of the locality loophole in which the measurement events themselves cannot influence each other by way of a light signal. Quoting a time resolution of under1ns would require the detectors to be at least 0.3m apart whereas in the experiment they were 2.5m apart. By incorporating high-speed switching of polariser directions during the flight of the photons as used by Aspect et al. [26] or Weihs et al. [27] the entanglement swapping Bell tests could also ensure *Strict Einstein separability*. One must point out that significance levels obtained using entanglement swapping are much lower than in more recent EPR experiments. Weihs et al. obtained a CHSH violation of 30 standard deviations. Surprisingly this does not seem to be due to losses in fidelity, because both Weih's and Kaltbaek's teams achieved a fidelity of 97%. Rather, it is simply due to fewer measurements being taken because of the longer experiment times and lower count rates [40]. In principle, there is no reason that entanglement swapping versions of Bell tests cannot achieve an equal significance.

3.2 Incorporating delayed-choice

A proof of principle was demonstrated by Jennewein et al. by passing Victor's photons through 10.5m lengths of fibre optic cables to delay the BSM until Alice and Bob had taken their measurements [18]. No loss of fidelity was observed which indicated that the $\left|\Psi^-\right\rangle$ state had indeed been teleported. If one wishes to give a realist interpretation to the quantum state of equation (2.1) then one is lead into the seeming paradox that a future measurement could influence past events

[11]. I shall return to the validity of this viewpoint shortly, however there is a much more mundane causal explanation that must first be considered - Alice and Bob's measurements could in fact be influencing the outcome of Victor's BSM. In order to rule this out as an explanation, Victor's BS and preferably his choice must be spacelike separated from Alice and Bob's measurement events.

In Jennewein's experiment, Victor's BS was spacelike separated from Alice and Bob's preventing the latter from influencing the former by means of a signal travelling at up to the speed of light. This is good evidence to support a *forward-in-time influence* hypothesis, however there was no 'choice' involved in this experiment. The delay was in place for the entire experiment allowing ample time for signals to pass from Alice and Bob's measuring apparatus to Victor's, and somehow conspire to 'fix' the results to appear as though entanglement was being projected backwards in time.

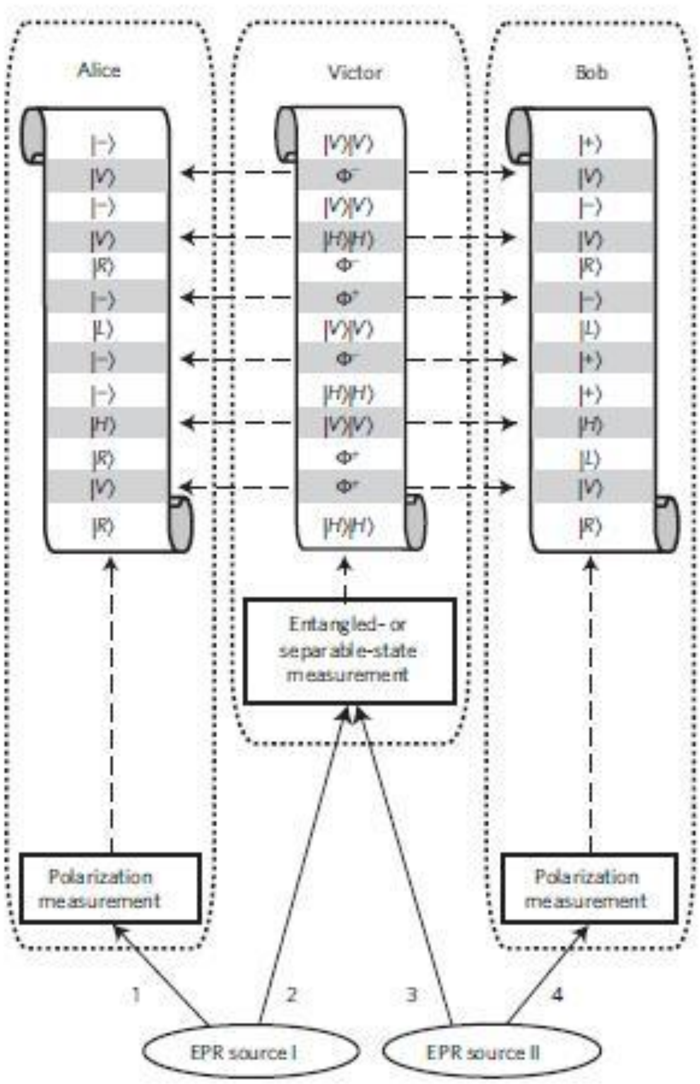

Figure 10 - Results from Victor's delayed-choice. Whenever he projects onto a Bell state (BSM), Alice and Bob's photons will show correlations corresponding to that state. Whenever he chooses to perform a SSM, Alice and Bob's photons are uncorrelated.

A more sophisticated experiment was conducted by Ma et al. [12] which employed random switching (the 'choice') between either a BSM or a separable state measurement[3] (SSM) being performed. Victor's choice was spacelike separated from Alice and Bob's measurement events which rules out either from having a causal influence on his results. The BSM could distinguish between the $\left|\Phi^{\pm}\right\rangle$ states which, by equation (2.1), means that Alice and Bob's photons were projected onto the corresponding states. Whenever a SSM was performed, Victor measured the polarisation of his photons directly and they were projected onto states that were not entangled, i.e. product states of the form $\left|\mathrm{H}\right\rangle_2 \left|\mathrm{H}\right\rangle_3$ etc. [50]. Alice and Bob, measuring at random in one of three mutually unbiased polarisation bases, then had an equal chance of measuring H or V polarisation in their chosen directions.

Ma's results agreed with Peres' prediction that dependent on Victor's choice and the outcome of his BSM, Alice and Bob could sort their measurements into subsets of photon pairs which displayed the appropriate correlations despite them having been observed earlier and their polarisations irrevocably recorded (see Figure10) [11]. The fidelity was 0.681 which meets criterion $\mathbb{B}2$ for showing entanglement. It is likely that the switching mechanism that they employed as Victor's choice introduced distinguishability between his photons which lead to the reduced fidelity compared with Jennewein's experiment. One hopes that this can be improved on in future experiments.

Although the fidelity was too low, one is naturally lead to question whether one can test Bell's theorem using delayed-choice entanglement. Given that entanglement swapping places greater restrictions on local realism [5, 7, 47], it would be interesting to investigate Bell's theorem in this circumstance. No such experiment has yet been performed using entangled pairs, however a recent experiment by Megidish et al. [13] is closely related but incorporates an intriguing twist – just as one could use delayed-choice to investigate entanglement between photons that had ceased to exist, the same principle can be used to project entanglement onto two photons that had never coexisted in time. Victor performs his BSM after Alice's photon has been detected and before Bob has detected his (see Figure 11). The wave function is still described by equation (2.1) despite the pairs being emitted at different times.

A fidelity of 0.77±0.01 was achieved which demonstrates ($\mathbb{B}2$) that Alice and Bob's photons are projected onto an entangled state despite neither coexisting. The author's attributed losses in fidelity to technical issues that should be improvable on in future experiments. Within experimental error at least, this just reaches the threshold required to observe a CHSH violation. A marginal violation was observed $\left(S_{\exp} = 2.04 \pm 0.04\right)$ which by itself is far from a definitive refutation of local realism. However, taken together with the results already discussed it is still tantalising evidence that we must either extend our concept of nonlocality to include time or abandon altogether a real interpretation of the quantum wave function.

---

[3] A separable state measurement is equivalent to directly measuring the polarisation of each photon. [*Ma 2012]

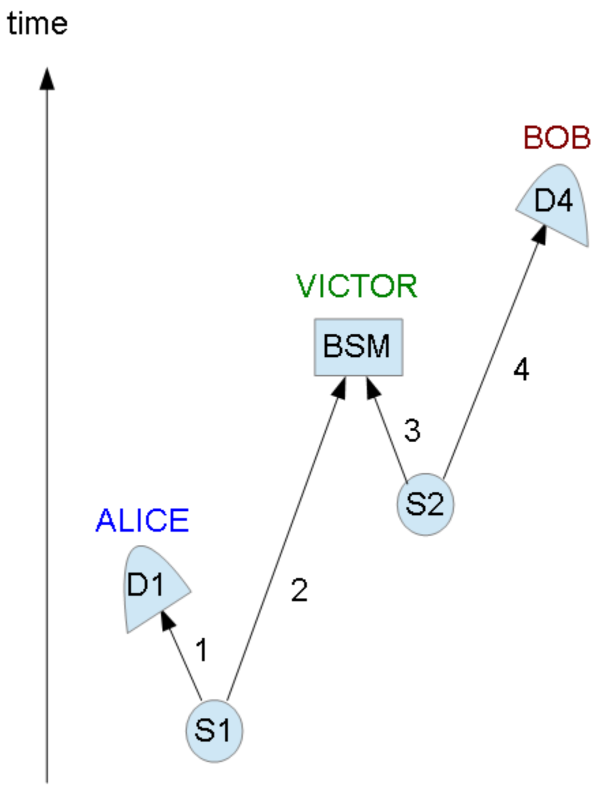

Figure 11 – Entanglement across time. A schematic procedure for establishing entanglement between photons 1 and 4, which never coexisted. These can then be used to test Bell's theorem.

How are we to make sense of this seeming paradox in the delayed-choice experiments? Is there a retrocausal effect? Not according to Peres, "[Only when] we attempt to attribute an objective meaning to the quantum state of a single system, [do] curious paradoxes occur…one has to clearly understand quantum mechanics and to believe in its correctness to see that there is no paradox" [11]. Using Bohr's concept of complementarity[4], Ma et al. viewed the BSM and the SSM as complementary measurements which are mutually exclusive in their ability to 'describe' Victor's photons [12]. Brukner et al. went further and showed that the amount of *information* that Victor obtained from measuring his photons was complementary with the amount of the *entanglement* produced [51]. As in other delayed-choice protocols [52, 53], the timing of the choice is irrelevant to the outcome of the experiment, which solely depends on the complementary measurement performed.

[4] Bohr proposed the concept of complementarity to account for mutually exclusive concepts such as *wave* and *particle* in quantum mechanics. Knowledge of one prevents us knowing the other [54].

Part 4 - Conclusions and outlook

Entanglement swapping provides us with an important additional tool with which to investigate the foundations of physics, one that is fundamentally different to any other. We have seen it used in testing Bell's theorem resulting in significant violations of the CHSH inequality (1.7) adding to the evidence against local realism, although all experiments so far have been affected by the locality and detector loopholes. Whilst not yet having the precision of other experiments, e.g. [27], there are no major technological reasons why this cannot be achieved. The production of the entangled state from truly independent sources is more challenging but has been demonstrated unequivocally [48]. Its use in testing local realism may not be far off and could be applied in a definitive loophole free test of local realism [7].

Delayed-choice entanglement swapping offers a unique way to study entanglement across time as well as spatial separations, something that the standard EPR experiment cannot offer. The seeming paradox created by the choice to entangle the photons being made after their detection can be explained using Bohr's complementarity principle and demonstrates the dangers of applying a real interpretation to the quantum state [6]. It is possible to test Bell's theorem using photons that have already been measured and the choice to entangle them lies in their future. Such an experiment has not yet been performed but we have seen a closely related version where the CHSH inequality was violated by photons that had never coexisted. Following on from [5], one may speculate whether Bell's theorem would continue to apply in such situations or might be a special case of a more general theory that might lead to greater conflicts with local realism in this new domain.

Entanglement swapping and its delayed-choice variant shed light on the nature of entanglement which is not yet properly understood. Phenomenon such as photon-vacuum entanglement [55] and counterfactual entanglement [56] suggest that entanglement may be more common than previously thought. It may well be that entanglement swapping and quantum teleportation are also more widespread. Further study should help elucidate entanglement and its relation to space and time.

References

[1] Yurke, B. and Stoler, D. (1992) 'Bell's-inequality experiments using independent-particle sources', *Physical Review A,* vol. 46, no. 5, pp. 2229-2234.

[2] Bouwmeester, D., Ekert, A.K. and Zeilinger, A. (eds) (2001) *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, 3rd edn. Germany, Springer-Verlag.

[3] Bennett, C., Brassard, G., Crepeau, C., Jozsa, R., Peres, A. and Wootters, W. (1993) 'Teleporting an Unknown Quantum State Via Dual Classical and Einstein-Podolsky-Rosen Channels', *Physical Review Letters,* vol. 70, no. 13, pp. 1895-1899.

[4] Pan, J., Bouwmeester, D., Weinfurter, H. and Zeilinger, A. (1998) 'Experimental entanglement swapping: Entangling photons that never interacted', *Physical Review Letters,* vol. 80, no. 18, pp.

[5] Bose, S., Vedral, V. and Knight, P. (1998) 'Multiparticle generalization of entanglement swapping', *Physical Review A,* vol. 57, no. 2, pp. 822-829.

[6] Bell, J.S. (2004) 'On the Einstein-Podolsky-Rosen paradox', in *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. Cambridge, Cambridge University Press.

[7] Greenberger, D.M., Horne, M., Zeilinger, A. and Zukowski, M. (2008) 'Bell’s theorem without inequalities for two particles. II. Inefficient detectors', *Physical Review A,* vol. 78, no. 2, pp. 022111.

[8] Greenberger, D.M., Horne, M.A., Shimony, A. and Zeilinger, A. (1990) 'Bell's theorem without inequalities', *American Journal of Physics,* vol. 58, no. 12, pp. 1131-1143.

[9] Clauser, J. and Shimony, A. (1978) 'Bells Theorem - Experimental Tests and Implications', *Reports on Progress in Physics,* vol. 41, no. 12, pp. 1881-1927.

[10] Pan, J., Bouwmeester, D., Daniell, M., Weinfurter, H. and Zeilinger, A. (2000) 'Experimental test of quantum nonlocality in three-photon Greenberger-Horne-Zeilinger entanglement', *Nature,* vol. 403, no. 6769, pp. 515-519.

[11] Peres, A. (2000) 'Delayed choice for entanglement swapping', *Journal of Modern Optics,* vol. 47, no. 2-3, pp. 139-143.

[12] Ma, X., Zotter, S., Kofler, J., Ursin, R., Jennewein, T., Brukner, C. and Zeilinger, A. (2012) 'Experimental delayed-choice entanglement swapping', *Nature Physics,* vol. 8, no. 6, pp. 478-483.

[13] Megidish, E., Halevy, A., Shacham, T., Dvir, T., Dovrat, L. and Eisenberg, H.S. (2013) 'Entanglement Swapping between Photons that have Never Coexisted', *Physical Review Letters,* vol. 110, no. 21, pp. 210403.

[14] Hardy, L. (1998) 'Spooky action at a distance in quantum mechanics', *Contemporary Physics,* vol. 39, no. 6, pp. 419-429.

[15] Hughes, R. (1992) '5.7 Composite Systems and Tensor-Product Spaces', in *The Structure and Interpretation of Quantum Mechanics*, First Harvard University Press paperback edition edn. United States of America, Harvard University Press.

[16] Bolton, J. and Mackintosh, R. (eds) *Quantum mechanics and its interpretation*, 2.1st edn., p. 150, United Kingdom, The Open University.

[17] Einstein, A., Podolsky, B. and Rosen, N. (1935) 'Can quantum-mechanical description of physical reality be considered complete?', *Physical Review,* vol. 47, no. 10, pp. 0777-0780.

[18] Jennewein, T., Weihs, G., Pan, J. and Zeilinger, A. (2002) 'Experimental nonlocality proof of quantum teleportation and entanglement swapping', *Physical Review Letters,* vol. 88, no. 1, pp. 017903.

[19] Clauser, J., Horne, M., Shimony, A. and Holt, R. (1969) 'Proposed Experiment to Test Local Hidden-Variable Theories', *Physical Review Letters,* vol. 23, no. 15, pp. 880-&.

[20] Bolton, J. and Mackintosh, R. (eds) *Quantum mechanics and its interpretation*, 2.1st edn., pp. 163-166, United Kingdom, The Open University.

[21] Bransden, B.H. and Joachin, C.J. (2007) '17.4 The problem with measurement', in *Quantum Mechanics*, 2nd edn. India, Dorling Kindersley (India) Pvt. Ltd.

[22] Aspect, A., Grangier, P. and Roger, G. (1982) 'Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment - a New Violation of Bell Inequalities', *Physical Review Letters,* vol. 49, no. 2, pp. 91-94.

[23] Bolton, J. and Mackintosh, R. (eds) *Quantum mechanics and its interpretation*, 2.1st edn., p. 162, United Kingdom, The Open University.

[24] Freedman, S. and Clauser, J. (1972) 'Experimental Test of Local Hidden-Variable Theories', *Physical Review Letters,* vol. 28, no. 14, pp. 938-941.

[25] Aspect, A., Grangier, P. and Roger, G. (1981) 'Experimental Tests of Realistic Local Theories Via Bells Theorem', *Physical Review Letters,* vol. 47, no. 7, pp. 460-463.

[26] Aspect, A., Dalibard, J. and Roger, G. (1982) 'Experimental Test of Bell Inequalities using Time-Varying Analyzers', *Physical Review Letters,* vol. 49, no. 25, pp. 1804-1807.

[27] Weihs, G., Jennewein, T., Simon, C., Weinfurter, H. and Zeilinger, A. (1998) 'Violation of Bell's inequality under strict Einstein locality conditions', *Physical Review Letters,* vol. 81, no. 23, pp. 5039-5043.

[28] Whitaker, M. (2011) '10. Alain Aspect: ruling out signalling', in *The New Quantum Age*, 1st edn. United States of America, Oxford University Press.

[29] Bohm, D. and Aharonov, Y. (1957) 'Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky', *Physical Review,* vol. 108, no. 4, pp. 1070-1076.

[30] Mandel, L. and Wolf, E. (1995) '4.2.1 Temporal coherence and the coherence time', in *Optical Coherence and Quantum Optics*, First edn. United States of America, Cambridge University Press.

[31] Zukowksi, M., Zeilinger, A., Horner, M. and Ekert, A. (1993) 'Event-Ready-Detectors Bell Experiment Via Entanglement Swapping', *Physical Review Letters,*vol. 71, no. 26, pp. 4287-4290.

[32] Mandel, L. and Wolf, E. (1995) '4.3.1 The laws of interference. The mutual coherence function and the complex degree of coherence', in *Optical Coherence and Quantum Optics*, First edn. United States of America, Cambridge University Press.

[33] Kaltenbaek, R., Prevedel, R., Aspelmeyer, M. and Zeilinger, A. (2009) 'High-fidelity entanglement swapping with fully independent sources', *Physical Review A,*vol. 79, no. 4, pp. 040302.

[34] Schumacher, B. and Nielsen, M. (1996) 'Quantum data processing and error correction', *Physical Review A,* vol. 54, no. 4, pp. 2629-2635.

[35] Ma, X. (2010) '*Nonlocal delayed-choice experiments with entangled photons'*, Unpublished thesis, University of Vienna.

[36] de Riedmatten, H., Marcikic, I., van Houwelingen, J., Tittel, W., Zbinden, H. and Gisin, N. (2005) 'Long-distance entanglement swapping with photons from separated sources', *Physical Review A,* vol. 71, no. 5, pp. 050302.

[37] Peres, A. (1996) 'Separability criterion for density matrices', *Physical Review Letters,* vol. 77, no. 8, pp. 1413-1415.

[38] Schmid, C., Kiesel, N., Weber, U.K., Ursin, R., Zeilinger, A. and Weinfurter, H. (2009) 'Quantum teleportation and entanglement swapping with linear optics logic gates', *New Journal of Physics,* vol. 11, pp. 033008.

[39] Zukowski, M., Zeilinger, A. and Weinfurter, H. (1995) 'Entangling Photons Radiated by Independent Pulsed Sources', *Fundamental Problems in Quantum Theory: a Conference Held in Honor of Professor John A.Wheeler,* vol. 755, pp. 91-102.

[40] Rarity, J. (1995) 'Interference of Single Photons from Separate Sources', *Fundamental Problems in Quantum Theory: a Conference Held in Honor of Professor John A.Wheeler,* vol. 755, pp. 624-631.

[41] Bell, J.S. (2004) 'Bertlmann's socks and the nature of reality' in *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. Cambridge, Cambridge University Press.

[42] Pavicic, M. 'Event-Ready Entanglement Preparation', *"Quantum Interferometry," Proceedings of the Adriatico Workshop, Trieste, March 1996. F. De Martini, G. Denardo, and Y. Shih (eds.), (VCH, New York, 1996), pp. 193-204*; available online at http://arxiv.org/abs/quant-ph/9907038

[43] Braunstein, S.L. and Kimble, H.J. (1998) 'A posteriori teleportation', *Nature,* vol. 394, no. 6696, pp. 840-841.

[44] Kok, P. and Braunstein, S. (2000) 'Entanglement swapping as event-ready entanglement preparation', *Fortschritte Der Physik-Progress of Physics,* vol. 48, no. 5-7, pp. 553-557.

[45] Pan, J., Chen, Z., Lu, C., Weinfurter, H., Zeilinger, A. and Zukowski, M. (2012) 'Multiphoton entanglement and interferometry', *Reviews of Modern Physics,* vol. 84, no. 2.

[46] Pan, J., Daniell, M., Gasparoni, S., Weihs, G. and Zeilinger, A. (2001) 'Experimental demonstration of four-photon entanglement and high-fidelity teleportation', *Physical Review Letters,* vol. 86, no. 20, pp. 4435-4438.

[47] Greenberger, D.M., Horne, M. and Zeilinger, A. (2008) 'Bell's theorem without inequalities for two particles. I. Efficient detectors', *Physical Review A,* vol. 78, no. 2, pp. 022110.

[48] Halder, M., Beveratos, A., Gisin, N., Scarani, V., Simon, C. and Zbinden, H. (2007) 'Entangling independent photons by time measurement', *Nature Physics,* vol. 3, no. 10, pp. 692-695.

[49] Yang, T., Zhang, Q., Chen, T., Lu, S., Yin, J., Pan, J., Wei, Z., Tian, J. and Zhang, J. (2006) 'Experimental synchronization of independent entangled photon sources', *Physical Review Letters,* vol. 96, no. 11, pp. 110501.

[50] Horodecki, R., Horodecki, P., Horodecki, M. and Horodecki, K. (2009) 'Quantum entanglement', *Reviews of Modern Physics,* vol. 81, no. 2, pp. 865-942.

[51] Brukner, C., Aspelmeyer, M. and Zeilinger, A. (2005) 'Complementarity and information in "delayed-choice for entanglement swapping"', *Foundations of Physics,* vol. 35, no. 11, pp. 1909-1919.

[52] Kim, Y., Yu, R., Kulik, S., Shih, Y. and Scully, M. (2000) 'Delayed "choice" quantum eraser', *Physical Review Letters,* vol. 84, no. 1, pp. 1-5.

[53] Jacques, V., Wu, E., Grosshans, F., Treussart, F., Grangier, P., Aspect, A. and Roch, J. (2007) 'Experimental realization of Wheeler's delayed-choice gedanken experiment', *Science,* vol. 315, no. 5814, pp. 966-968.

[54] Bransden, B.H. and Joachin, C.J. (2007) '2.1 Wave-particle duality', in *Quantum Mechanics*, 2nd edn. India, Dorling Kindersley (India) Pvt. Ltd.

[55] Sciarrino, F., Lombardi, E., Milani, G. and De Martini, F. (2002) 'Delayed-choice entanglement swapping with vacuum-one-photon quantum states', *Physical Review A,* vol. 66, no. 2, pp. 024309.

[56] Cohen, O. (1999) 'Counterfactual entanglement and nonlocal correlations in separable states', *Physical Review a,* vol. 60, no. 1, pp. 80-84.

[57] Bell, J.S. (2004) 'de Broglie-Bohm, delayed-choice double-slit experiment, and density matrix', in *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. Cambridge, Cambridge University Press.